Module 7
Solving Complex Problems

SCRATCH

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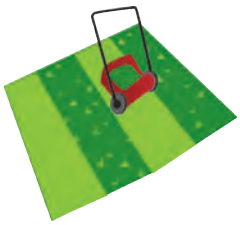
The Towers of Hanoi

Linear Complexity



- Mowing the lawn is a problem that demonstrates linear complexity.
- If you double the size of the area that you want to mow, it will take twice as long to complete the task.

More Complex Problems

- Many problems will increase in complexity very quickly each time an extra input is added.
- Explore the Towers Of Hanoi Puzzle online at http://www.tinyurl.com/hanoi1

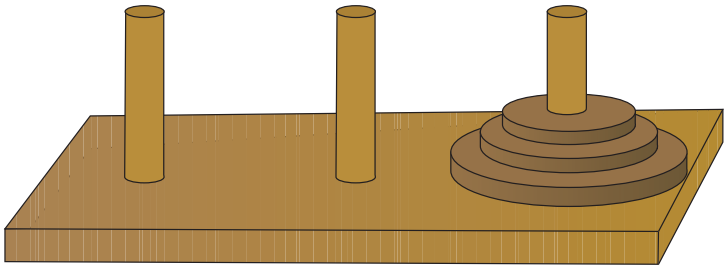
The instructions button on the web page describes the rules of the puzzle. The goal of the towers of Hanoi problem is to move a stack of disks from one peg to another in as few moves as possible. You may move only one disk at a time. You may not stack larger disks on top of smaller disks.

The puzzle was invented by the French mathematician Édouard Lucas in 1883. There is a legend about a Vietnamese or Indian temple which contains a large room with three time-worn posts surrounded by 64 golden disks. The priests of Brahma, acting out the command of an ancient prophecy, have been moving these disks, in accordance with the rules of the puzzle. According to the legend, when the last move of the puzzle is completed, the world will end. It is not clear whether Lucas invented this legend or was inspired by it.

How many moves does it take?

Did you know you should be able to complete the puzzle in $(2^n -1)$ moves where n is the number of disks?
For 3 disks $2^n -1 = (2^3 – 1) = (8-1) = 7$ moves

The number of moves increases significantly as more disks are added.
In mathematics the phrase exponential growth is used to describe the increase in the number of moves required to solve the problem in relation to the number of disks



If the legend of the Vietnamese or Indian temple were true, and if the priests were able to move disks at a rate of one per second, using the smallest number of moves, it would take them roughly 272 thousand years to complete.

Exercises

❶ See if you can use your maths skills to complete the table below that details the growth in complexity for the Towers of Hanoi problem.

Remember it takes (2^n-1) moves where n=number of disks.

Number of Disks	Number of Moves Required to Solve the Problem
6	63
9	511
15	
25	
	1,099,511,627,775
64	9,223,372,036,854,775,808

❷ *Download the "Towers of Hanoi" Scratch Project in the resources section of* ***Module 7*** *at http://www.scratch.ie/students*

Once you have the project open in Scratch click on the 'Show project notes' button under the File menu to view instructions for the game. Take a look at the script for the game. Then play the game and see if you can modify the script to make improvements.

The Travelling Salesman Problem

The Travelling Salesman Problem is one of the most intensively studied problems in computational mathematics. These pages are devoted to the history, applications, and current research of this challenge of finding the shortest route visiting each member of a collection of locations and returning to your starting point.

The Travelling Salesman Problem

- For a given set of cities, visit each city once and minimise the distance you travel.
- Explore the puzzle http://www.tinyurl.com/salesman1

How many possible routes?

- Remember it takes (n-1)! moves where n=number of cities
- E.G. For 10 cities (n-1)! = (10-1)! = 9! = 9x8x7x6x5x4x3x2x1

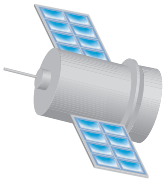
Number of Cities	Number of Possible Routes
10	362, 880
12	39,916,800
15	87,178,291,200
25	620,448,401,733,239,439,360,000

How much time does it take?

- These calculations are based on a computer being able to execute 1 million instructions per second.
- The problem cannot be solved in a reasonable amount of time for 25 cities. This does not mean that the problem cannot be solved for 25 cities. Computer science researchers have devised new approaches besides analysing all possible routes to solving the Travelling Salesman Problem. They use optimisation and approximation techniques to find a near-optimal solution.

Number of Cities	Number of Possible Routes
10	0.36 seconds
12	39.91 seconds
15	24.21 hours
25	196 billion years

Not just for The Travelling Salesman



- Although transportation applications are the most natural setting for the Travelling Salesman Problem, the model has led to many interesting applications in other areas.
- Used in Biology to compute DNA sequences.
- A Travelling Salesman algorithm is used to minimise the use of fuel in targeting and imaging manoeuvres for the pair of satellites involved in NASA Starlight space interferometer program.

Exercises

❶ A band wishes to tour ten destinations across the Southern region of Ireland that have been circled in the map below over two weeks during the summer to promote the release of their new album. The tour will start and finish in Dublin. With the help of the map and the legend indicating distances between the towns, fill the table below to indicate the shortest round trip to Dublin that the band can take during their tour.

Distances in Km	Clonmel	Cork	Dublin	Kilkenny	Limerick	Portlaoise	Thurles	Tipperary Town	Tralee	Waterford
Clonmel		100	170	45	80	110	55	40	170	55
Cork	100		250	155	100	165	110	95	120	120
Dublin	170	250		125	195	85	140	175	295	160
Kilkenny	45	155	125		125	45	50	85	225	60
Limerick	80	100	195	125		110	75	45	110	130
Portlaoise	110	165	85	45	110		65	95	210	100
Thurles	55	110	140	50	75	65		45	180	90
Tipperary Town	40	95	175	85	45	95	45		140	85
Tralee	170	120	295	225	110	210	180	140		205
Waterford	55	120	160	60	130	100	90	85	205	

Date	Journey From	Journey To	Distance
Jun-21	Dublin		
Jun-22			
Jun-23			
Jun-24			
Jun-25			
Jun-26			
Jun-27			
Jun-28			
Jun-29			
Jun-30		Dublin	
		Total Distance	

End of Module Quiz

1. In the Travelling Salesman Problem the growth of possible routes in relation to the number of locations that you wish to visit is known as

    A. Linear Growth
    B. Exponential Growth
    C. Factorial Growth
    D. Power Growth

2. If it takes one second to process each move, how long will it take a computer in hours minutes and seconds to solve the Towers of Hanoi in the least moves possible for twelve disks?
Remember it takes (2^n-1) moves where n=number of disks.

    A. 1 hour 8 minutes 16 seconds
    B. 17 seconds
    C. 34 minutes 8 seconds
    D. 1 hour 8 minutes 15 seconds

3. Computer science researchers have devised approaches to solve large complex problems. Which one of these methods is most suitable for solving such problems?

    A. Optimisation and approximation algorithms
    B. Exhaustive algorithms
    C. Brute force algorithms
    D. Linear algorithms

4. Which one of these is NOT a complex problem?

    A. Colouring a map of European countries with 4 different colours
    B. Travelling Salesman Problem
    C. Calculating the area of a rectangle
    D. Timetabling for a School

5. How many possible routes are there for a nine cities Travelling Salesman Problem? Remember there are (n-1)! possible routes where n=number of cities.

    A. 8
    B. 40320
    C. 16777216
    D. 362880